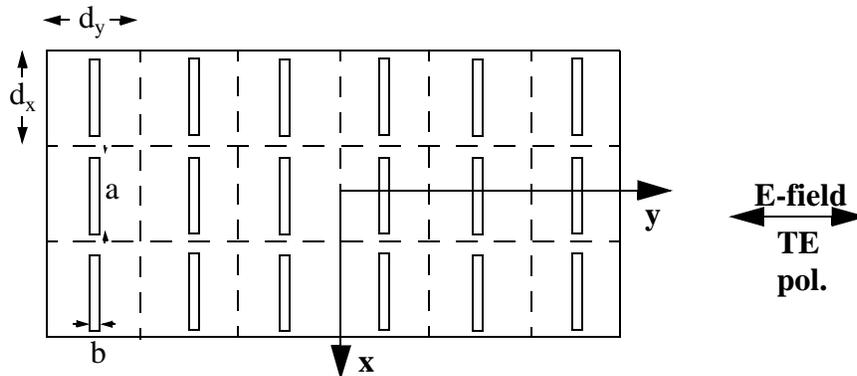


# Using Periodicity to Control Spectral Characteristics of an Array of Narrow Slots

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In this paper we consider a frequency selective surface (FSS) consisting of a thin perfectly conducting sheet periodically perforated with narrow slot apertures. For wavelengths greater than the periodicity, such a surface performs similarly to a bandpass filter since the periodic elements within the FSS possess resonance characteristics. The effects on the spectral characteristics of varying the periodicity of a narrow slot array (or its complementary dipole array) was studied by Ott et al<sup>1</sup> and Munk et al<sup>2</sup>. By changing the periodicity, they found it possible to change the location of the resonant wavelength and the bandpass width of the transmission/reflection spectrum. With advances in computational technology, we are able in this paper to study these effects of periodicity in narrow slot arrays in much more detail. In this study we used the methods described by Chen<sup>3</sup> to predict the spectral characteristics of narrow slot arrays designed for use in the middle infrared spectral region. An algorithm was written in FORTRAN for use on either a PC or workstation. Although our work presented here is limited to a normally incident plane wave, the approach can be extended to the case of oblique incidence.

The geometry of the narrow slot array is shown in figure 1. For this study, the slot dimensions were kept constant with  $a=5.0\mu\text{m}$  and  $b=0.5\mu\text{m}$ , while the periodicity in the  $x$  and  $y$  directions were varied between  $6.0\mu\text{m}$  and  $12.0\mu\text{m}$  in  $1.0\mu\text{m}$  increments. Polarization for the normally incident plane wave is limited to TE, where the E-field is perpendicular to the long dimension of the slots, as to produce a resonance effect in the slot. As stated by Chase and Joseph<sup>4</sup>, a *single* narrow slot



**Figure 1** - Geometry used for Frequency Selective Surface consisting of a narrow slot array.

will produce a transmission resonance at

$$\lambda_{res} = 2.1 \left( 1 + \frac{b}{2a} \right) a. \quad (1)$$

Thus for a single slot the resonance wavelength is  $11.025\mu\text{m}$ . Coupling between the slots in the array perturbs the resonance wavelength value from this value depending on the periodicity of the array.

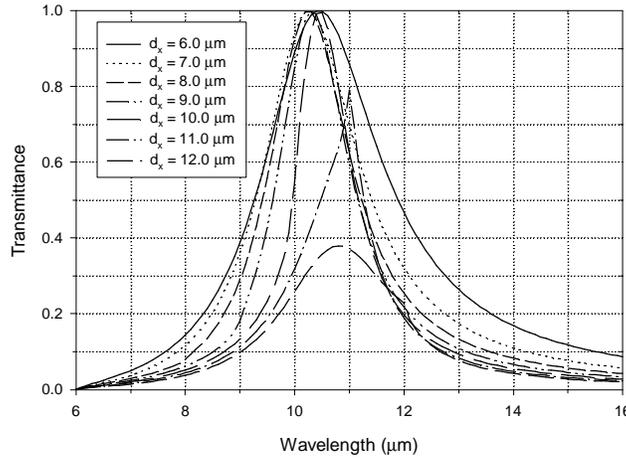
In the first case considered we examine the effects of varying  $d_x$  from  $6.0\mu\text{m}$  to  $12.0\mu\text{m}$  in  $1.0\mu\text{m}$  increments while  $d_y$  is held constant at  $6.0\mu\text{m}$ . The resulting transmission profiles are presented in figure 2. It should be noted that the diffraction edge,  $\lambda_d$ , is equal to  $d_x$ . For wavelengths larger than  $\lambda_d$ , only the zeroth diffraction order propagates. At wavelengths less than the period in the  $x$ -direction,  $d_x$ , multiple diffraction orders are propagating (though only the zeroth order is shown in figure 2). From figure 2 it is clear that as  $d_x$  increases (thus increasing  $\lambda_d$ ), the diffraction edge ‘‘cuts into’’ the resonance profile. However, the resonance and bandpass character are not drastically affected by varying the period in this direction.

For the next case, we vary  $d_y$  from  $6.0\mu\text{m}$  to  $12.0\mu\text{m}$  in  $1.0\mu\text{m}$  increments while  $d_x$  is held constant at  $6.0\mu\text{m}$ . Figure 3 shows the resulting transmission profiles. For this example, the diffraction edge corresponds to the value of  $d_y$  instead of  $d_x$ . Unlike the previous case, varying the period in the  $y$ -direction (the direction of the E-field) changes both the resonant wavelength and the bandpass width.

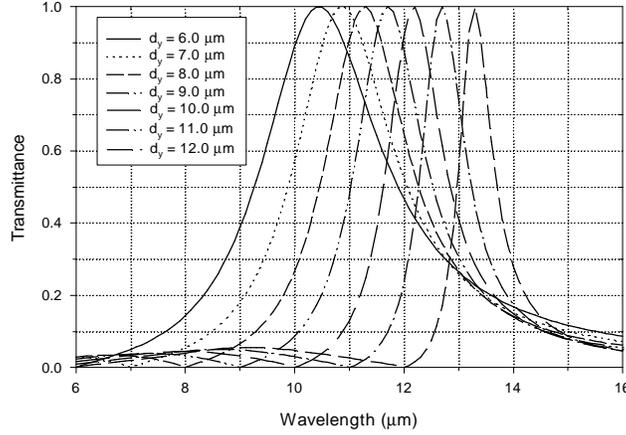
To help quantify these variations in resonant wavelength and bandpass width, the values were fitted using the general least squares method to the exponential equation

$$y = a + b \exp\left(-\frac{x}{c}\right), \quad (2)$$

where  $a$ ,  $b$ , and  $c$  are the parameters to be fit. The results are presented in table 1.



**Figure 2** - Transmission profiles for FSS in figure 1 with  $a=5.0\mu\text{m}$ ,  $b=0.5\mu\text{m}$ ,  $d_x$  varies from  $6.0\mu\text{m}$  to  $12.0\mu\text{m}$ ,  $d_y=6.0\mu\text{m}$ . The normally incident plane wave is TE polarized.



**Figure 3** - Transmission profiles for FSS in figure 1 with  $a=5.0\mu\text{m}$ ,  $b=0.5\mu\text{m}$ ,  $d_x=6.0\mu\text{m}$ ,  $d_y$  varies from  $6.0\mu\text{m}$  to  $12.0\mu\text{m}$ . The normally incident plane wave is TE polarized.

The error measurement,  $StdErr$ , listed in table 1 is the root mean square error, found by

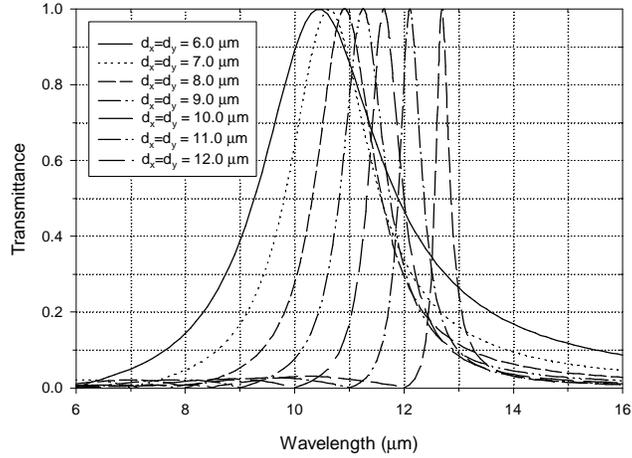
$$StdErr = \sqrt{\frac{\chi^2}{DegreesofFreedom}}. \quad (3)$$

In the final case under consideration, we varied both  $d_x=d_y$  from  $6.0\mu\text{m}$  to  $12.0\mu\text{m}$  in  $1.0\mu\text{m}$  increments. The transmission profiles are shown in figure 4. Again, the resonant wavelength,  $\lambda_r$ , and the bandpass width,  $\Delta\lambda$ , were fitted to an exponential, using the same procedure as in the previous example. Since variation of the periodicity only in the  $x$ -direction (figure 2) produced little effect on  $\lambda_r$  and  $\Delta\lambda$ , it should be expected that variation of both  $d_x=d_y$  (figure 4) should produce results similar to that of the variation of  $d_y$  alone (figure 3). Comparisons of figures 3 and 4 show that effects of the two variations are similar, but not exactly the same. Examination of the parameter values in table 1 confirms this.

Varying both  $d_x$  and  $d_y$  together produces a more pronounced non-linear increase in both  $\lambda_r$  and  $\Delta\lambda$  than varying just  $d_y$  alone. In fact, the variation in  $\lambda_r$  as  $d_y$  changes is nearly linear in nature. While the relationship between  $\Delta\lambda$  vs.  $d_y$  for

**Table 1: Parameters fitted to equation  $y = a + b \exp(-x/c)$**

Figure	y	x	a	b	c	StdErr
Figure 3	$\lambda_r$	$d_y$	5.5418	3.1377	-13.3097	0.0228
	$\Delta\lambda$	$d_y$	-0.8481	7.5578	7.6991	0.0149
Figure 4	$\lambda_r$	$p (=d_x=d_y)$	9.5337	0.2653	-4.8386	0.0193
	$\Delta\lambda$	$p (=d_x=d_y)$	0.0454	22.8311	2.7417	0.0440



**Figure 4** - Transmission profiles for FSS in figure 1 with  $a=5.0\mu\text{m}$ ,  $b=0.5\mu\text{m}$ ,  $d_x=d_y$  varies from  $6.0\mu\text{m}$  to  $12.0\mu\text{m}$ . The normally incident plane wave is TE polarized.

the case of varying  $d_y$  alone is not quite as linear as for  $\lambda_r$  vs.  $d_y$ , it is much closer to being a linear relationship than for the corresponding case of varying both  $d_x$  and  $d_y$  together. To confirm this, one need only compare the ratio of the third and second terms in the expansion of equation (2),  $\frac{1}{2|c|}$ , for each case in question. If the condition

$$\frac{1}{2|c|} \ll 1, \quad (4)$$

occurs, then the exponential relationship is approaching a linear one. In the  $\lambda_r$  case described in figure 3, this ratio is 0.003, while for the case in figure 4, it is 0.021, or nearly a factor of ten. In the  $\Delta\lambda$  case, these values are 0.065 (figure 3) and 0.182 (figure 4), or a factor of nearly 3.

In conclusion, establishment of a more explicit relationship between  $d_y$  and  $\lambda_r$ , and  $d_y$  and  $\Delta\lambda$ , make periodicity variations in the  $y$ -direction an attractive way to “fine tune” the location of the resonant wavelength and bandpass width when designing a narrow slot array FSS. These design parameters become important when trying to account for design specifications of infrared filters.

## **References**

1. R. H. Ott, R. G. Kouyoumjian, and L. Peters, Jr., “Scattering by a Two-Dimensional Periodic Array of Narrow Plates”, *Radio Science*, **2**, 1347-1359 (1967).
2. B. A. Munk, R. G. Kouyoumjian, and L. Peters. Jr., “Reflection Properties of Periodic Surfaces of Loaded Dipoles”, *IEEE Trans. Antennas Propagat.*, **AP-19**, 612-617 (1971).
3. C. C. Chen, “Transmission Through a Conducting Screen Perforated Periodically with Apertures”, *IEEE Trans. Microwave Theory Tech.*, **MTT-18**, 627-632 (1970).
4. S. T. Chase and R. D. Joseph, “Resonant Array Bandpass Filters for the Far Infrared”, *Applied Optics*, **22**, 1775-1779 (1983).